The value of the test rests upon the facility with which the multiple of 21 is created, inasmuch as to multiply by 2 is a mental process much more surely within the mind of a child than dividing by 7.

2. A test for the divisibility by 13 is to be found by a similar process and upon the same principles—being based on the fact that 91 is a multiple of 7. Multiply the unit figure by 9 and find the difference between the product and the number without its unit figure. Thus, in 1183, $9\times3=27$, $118-27=91$, and in 91, $9\times1=9$, $9-9=0$. For 325, $9\times5=45$, $45-32=13$.

3. Likewise for 17, multiply by 5, since 51 is a multiple of 17. So for 595, $5\times5=25$; $59-24=34$. For 2244, $5\times4=20$, $224-20=204$; $5\times4=20$, $20-20=0$.

NOTE ON THE EVOLUTE OF AN ALGEBRAIC CURVE.

By A. H. WILSON, Instructor of Mathematics, University of Illinois.

The following method of forming the evolute of an algebraic curve may be of interest.

Let $f(x, y)=\varphi$ represent the curve, and $y-y_1=l(x-x_1)$ its normal at the point $(x_1 y_1)$ on the curve, l being a function of x_1 and y_1. The elimination of x_1 (or y_1) between $f(x_1, y_1)=0$ and $\beta-y_1=l(\alpha-x_1)$, gives an equation

$$\varphi(y_1)=0 \text{ (or } \psi(x_1)=0),$$

whose roots are the ordinates (or the abscissas) of the points on the curve the normals at which pass through the point (α, β).

The evolute may be regarded as the locus of points from which two of the normals through (α, β) to the curve are coincident; and hence the equation of the evolute is the relation between α and β obtained by setting equal to zero the discriminant of $\varphi=0$ (or $\psi=0$).

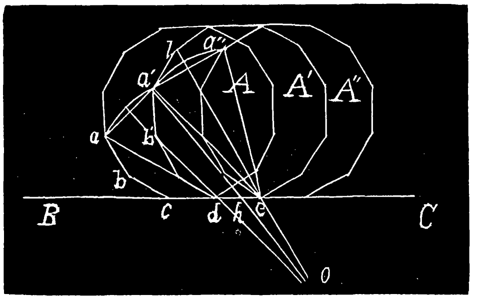
The application of the method is obviously very limited.

DETERMINATION OF THE RADIUS OF CURVATURE OF THE CYCLOID WITHOUT THE AID OF THE CALCULUS.

By FREDERIC R. HONEY, Hartford, Conn.

Let A represent any regular polygon. If we roll it along the straight line BC into the positions A', A'', ... bringing each side in succession into coinci-

dence with it, any angular point as a, will trace a series of arcs of circles aa', $a'a''$, ... whose centers will be on the line BC. The distance between two consecutive centers will be equal to the side of the polygon. The arc aa' will be described with the radius da; $a'a''$ with the radius ea', ...



The center o of an arc $aa'a''$ which will pass through the points a, a', a'', will be at the intersection of the bisector of the angle ada' and the bisector of the angle $a'ea''$. Since the polygon is inscriptible, the angle $a'el$ is equal to the alternate angle $ea'd$. Therefore, leo is parallel to $a'd$. Similarly, $b'do$ is parallel to $a'e$. Therefore, $a'eod$ is a parallelogram. Its diagonal oa', the radius with which the arc $aa'a''$ is described, is bisected at h.

The above demonstration is applicable to a regular polygon with *any* number of sides. We will now suppose that the number is increased. The length of the side diminishes, and the points d, h, and e approach each other. At the limit, when the polygon becomes a circle, they coalesce, and ha' is the normal. The broken curved line $aa'a''$.......... becomes a cycloid, and the radius of curvature $oa'=2ha'$.

REMARKS ON DEFINITIONS IN TEXT-BOOKS ON GEOMETRY.

By G. W. GREENWOOD, M. A., McKendree College.

What is a circle? A common definition is that it is a plane figure—or a portion of a plane—bounded by a curved line, every point of which is equally distant from a point *within*, called the center. This gives us the impression that a circle is a disc, whereas in more advanced work it is regarded, with other conic sections, merely as a plane curve. It would be better if it were so defined in elementary texts. When we turn, however, to some texts which define it thus, we find something like this: 'A circle is a plane *closed line*, such that all straight lines joining *any* point on this line to a *certain point within* the figure, are equal.'

I shall endeavor to show that such a definition, like the more common one first given, is by no means logical. For this purpose, let us compare them with the following, which, while not altogether free from objections, is, I believe, logical: The locus of points in a plane, at congruent distances from a fixed point in the plane, is a circle. It will be noticed here that nothing is stated or implied concerning the form of the locus or any other properties save the one stated. The word *within* in the earlier definition is unnecessary unless, like a vermiform appendix, it indicates the evolution of this definition from the one first